The Prince - or better no prince?
The strategic value of appointing a successor

Online Appendix

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This appendix proves Proposition 4. Before we solve the game in Section 4 for an equilibrium, we recall the assumptions we make regarding the parameters that allow for interior solutions and that have been shown to be compatible with one another:

\[ a \in (1, \sqrt{6} - 1), \, \beta \in (1/a, 1) \text{ and } \lambda \in (4/(5 - a^2), 2/a). \]

Solving the two-period finite-horizon game backward, we directly find the period-1 payoffs of players. The player who is the king in period 1 receives the governance rent of this period with a period-0 present value that was normalized to \( \delta G_1 = 1 \). All other players receive zero. This applies in both regimes.

Turn next to period 0. Consider first the no-prince regime. One, randomly chosen duke has an opportunity to revolt. If he revolts, he must choose his revolt effort. Our parameter restrictions make sure that the optimal choice \( e^N \in [0, a/2] \) is determined by

\[ e^N = \arg \max_{e \in [0, a/2]} \left\{ f^N \cdot 1 - e \right\} = \frac{a - 1}{2}. \] (A1)

From (12) in the text, this leads to \( f^N = \frac{a^2 - 1}{4} \). Our parameter restrictions make sure that \( f^N \) is in the interval \((0, 1)\). The revolting duke gets a payoff of

\[ f^N - e^N = \left( \frac{a - 1}{2} \right)^2. \] (A2)

This is the minimum that the king needs to offer each duke to prevent a revolt. This gives the king an expected payoff in the best peaceful equilibrium of

\[ G_0 + 1 - 2 \left( \frac{a - 1}{2} \right)^2. \] (A3)

Turn next to the regime with a prince. We determine the minimum payment offers \((p, s)\) needed to achieve a peaceful outcome. Consider first the optimal effort choices in the various scenarios. If the prince does not revolt, then the duke needs to overthrow an executive government that consists of both the king and the prince. Given our assumptions about \( a \) and \( \beta \), the duke's effort choice is

\[ e^s = \arg \max_{e \in [0, a/2]} \left\{ f^s \cdot 1 - e \right\} = \frac{\beta a - 1}{2\beta}. \] (A4)

Here we need the assumption \( \beta > 1/a \) for an interior solution, and \( \beta < 1 \) reflects the assumption of a barrier effect. This effort determines the revolt success probability of the duke as \( f^s = (\beta^2 a^2 - 1)/(4\beta) \). Furthermore, our parametric assumptions imply that

\(^1\)It is straightforward to verify that the objective functions in the optimization problems in (A1), (A4), and (A9) are all concave. Therefore, in all these optimization problems, the effort choices determined by the first-order-conditions are indeed optimal.
\[ f^* = \frac{\beta^2 a^2 - 1}{4\beta} < \frac{\beta^2 5 - \beta^2}{4\beta} = \beta < 1 \]

and \( f^* \) is obviously positive. This gives the revolting duke an expected payoff

\[ f^* \cdot 1 - e^* = \frac{(\beta a - 1)^2}{4\beta}. \quad (A5) \]

In order to prevent the duke from revolting, the king needs to offer at least

\[ s = \frac{(\beta a - 1)^2}{4\beta} \quad (A6) \]

Next, suppose the prince revolts. In this case, similar to the stationary framework with infinite horizon, whether the coup fails or succeeds, the duke will revolt. This is important to know for the prince, as the prince faces a revolt should his own revolt be successful. This prospect of facing the revolt by the duke reduces the value of overthrowing the king for the prince. This value is

\[ 1 \cdot (1 - f^N) = 1 - \frac{a^2 - 1}{4} = \frac{5 - a^2}{4} = W. \quad (A7) \]

Note that because \( 0 < f^N = \frac{a^2 - 1}{4} < 1, 0 < W < 1 \). This observation, together with (A7), implies that \( \lambda \in (4/(5 - a^2), 2/a) \) is equivalent to

\[ 1 < \frac{1}{W} = \frac{4}{(5 - a^2)} < \lambda < \frac{2}{a}. \quad (A8) \]

When the prince revolts, his optimal effort is

\[ e^p = \arg \max_{e \in [0, \frac{a}{2}]} \{(\lambda a e - \lambda e^2) \cdot \left(\frac{5 - a^2}{4}\right) - e\} = \frac{1}{2} \frac{\lambda a W - 1}{\lambda W}. \quad (A9) \]

From \( a \in (1, \sqrt{6} - 1) \), and \( \lambda \in (4/(5 - a^2), 2/a) \), we know that \( \lambda > \frac{1}{W} > \frac{1}{\alpha W} \), so \( e^p \in (0, \frac{a}{2}) \).

Inserting (A9) in (13) in the text yields

\[ \pi_p = \frac{\lambda^2 a^2 W^2 - 1}{4\lambda W^2}. \]

Furthermore, (A8) implies that

\[ \pi_p = \frac{\lambda^2 a^2 W^2 - 1}{4\lambda W^2} < \frac{\lambda^2 a^2 W^2 - 1}{4W^2} < \frac{\lambda^2 a^2 W^2}{4W^2} = \frac{\lambda^2 a^2}{4} < 1, \]

and \( \pi^p \) is obviously positive. The prince, hence, receives a payoff \( \pi^p \cdot W - e^p \) from revolting, which can be written as
Accordingly, this is what the king needs to offer the prince to avoid a revolt. Overall this turns the payoff of the king into

\[ G_0 + 1 - p - s = G_0 + 1 - \frac{1}{4} \frac{(\lambda \alpha W - 1)^2}{\lambda W} - \frac{(\beta a - 1)^2}{4\beta}. \]  

(A11)

We now compare (A2) to (A6). Using \( a \in (1, \sqrt{6} - 1) \), and \( \beta \in (1/a, 1) \), we get \( \beta > \frac{1}{a} > \frac{1}{\sqrt{\alpha}} \), or \( \beta \alpha^2 > 1 \). Noting that \( (1 - \beta) > 0 \), this implies \( (1 - \beta) < \beta a^2 (1 - \beta) \), which in turn implies

\[ \beta^2 a^2 - 2\beta a + 1 < \beta a^2 - 2\beta a + \beta \implies \frac{(\beta a - 1)^2}{4\beta} < \frac{(a - 1)^2}{4}. \]  

(A12)

The appointment of a prince reduces the duke’s payoff. This is the barrier effect.

We now compare (A2) to (A10). Using \( a \in (1, \sqrt{6} - 1) \) and (A8), we get \( \lambda > \frac{1}{W} > \frac{1}{\sqrt{\alpha} W} \), which implies \( \lambda a^2 W > 1 \). Noting that \( (1 - \lambda W) < 0 \), we get \( (1 - \lambda W) \lambda a^2 W < (1 - \lambda W) \). This in turn implies

\[ \lambda a^2 W - 2\lambda W a + \lambda W < \lambda^2 a^2 W^2 - 2\lambda W a + 1 \implies \frac{(a - 1)^2}{4} < \frac{(\lambda a W - 1)^2}{4\lambda W}. \]  

(A13)

The king needs to offer the prince a higher compensation than when he was a duke under the no-prince regime. This is the elevated threat effect.

We may now compare the king’s payoffs in the two regimes. The king is better-off under the prince regime if

\[ \frac{(\lambda a W - 1)^2}{4\lambda W} + \frac{(\beta a - 1)^2}{4\beta} < 2\frac{(a - 1)^2}{4}. \]